Optimal Design of Inductive Components Based on Accurate Loss and Thermal Models

Dr. Jonas Mühlethaler
Introduction

System Layer
(GeckoCIRCUITS)

Component Layer
(GeckoMAGNETICS)
Introduction

GeckoMAGNETICS

How are magnetic components modeled in GeckoMAGNETICS?

(Gecko-Simulations; www.gecko-simulations.com)
Introduction

System Layer

Component Layer

Material Layer

www.ferroxcube.com
Introduction
Application of Inductive Components (1) : Buck Converter
(DC Current + HF Ripple)

Modeling Difficulties
- Non-sinusoidal current / flux waveform
- Current / flux is DC biased

Solutions
- FFT of current waveform for the calculation of winding losses
- Determine core loss energy for each segment and for each corner point in the piecewise-linear flux waveform
- Loss Map enables to consider a DC bias
Introduction
Application of Inductive Components (2) : Inductor of DAB Converter
(Non-Sinusoidal AC Current)

Schematic

Current / Flux Waveform

Modeling Difficulties
- Non-sinusoidal current / flux waveform
- Core losses occur in the interval of constant flux

Solutions
- FFT of current waveform for the calculation of winding losses
- Improved core loss equation that considers relaxation effects
Introduction
Application of Inductive Components (3) : Three-Phase PFC
(Sinusoidal Current + HF Ripple)

Schematic

Modeling Difficulties
- Non-sinusoidal current / flux waveform
- Major loop and many (DC biased) minor loops

Solutions
- FFT of current waveform for the calculation of winding losses
- Determine core loss energy for each segment and for each corner point in the piecewise-linear flux waveform (-> minor loop losses)
- Add major loop losses
Introduction
Overview of Different Flux Waveforms

Sinusoidal

DC Current + HF Ripple

Non-Sinusoidal AC Current

Sinusoidal Current + HF Ripple

Minor Loop

Major Loop
Introduction

System Layer

Component Layer

Material Layer
Introduction
Overview of Other Modeling Issues
Introduction
Wide Range of Realization Options

Inductors / Transformers

Core Shapes

Conductor Shapes


Introduction
Modeling Inductive Components (1)

Procedure

1) A reluctance model is introduced to describe the electric/magnetic interface, i.e. \( L = f(i) \).

\[ \downarrow \]

2) Core losses are calculated.

\[ \downarrow \]

3) Winding losses are calculated.

\[ \downarrow \]

4) Inductor temperature is calculated.
The following effects are taken into consideration:

**Magnetic Circuit Model (e.g. for Inductance Calculation):**
- Air gap stray field
- Non-linearity of core material

**Core Losses:**
- DC Bias
- Different flux waveforms (link to circuit simulator)
- Wide range of flux densities and frequencies
- Different core shapes

**Winding Losses:**
- Skin and proximity effect
- Stray field proximity effect
- Effect of core on magnetic field distribution
- Litz, solid, and foil conductors
Introduction

System Layer

Component Layer

Material Layer

Heat Dissipation

Core Shape / Material

Air Gap Stray Field

Winding Type / Arrangement

www.ferroxcube.com
Sometime there are parameters that bring advantages for one subsystem while deteriorating another subsystem (e.g. frequency in above example).
In order to get an optimal system design, an overall system optimization has to be performed.

It is (often) not enough to optimize subsystems independent of each other.
Outline

- Magnetic Circuit Modeling
- Core Loss Modeling
- Winding Loss Modeling
- Thermal Modeling
- Multi-Objective Optimization
- Summary & Conclusion
Magnetic Circuit Modeling
Reluctance Model

Electric Network
Magnetic Network

Conductivity / Permeability

\( \kappa \quad \mu \)

Resistance / Reluctance

\( R = \frac{l}{(\kappa A)} \quad R_m = \frac{l}{(\mu A)} \)

Voltage / MMF

\( V = \int_{P_1}^{P_2} \bar{E} \, d\bar{s} \quad V_m = \int_{P_1}^{P_2} \bar{H} \, d\bar{s} \)

Current / Flux

\( I = \iint_{A} \bar{j} \, d\bar{A} \quad \Phi = \iint_{A} \bar{B} \, d\bar{A} \)
Magnetic Circuit Modeling
Why a Reluctance Model is Needed

A reluctance model is needed in order to

calculate the inductance \( L = \frac{N^2}{R_{\text{tot}}} \)
calculation the saturation current
calculate the air gap stray field
calculate the core flux density

Accurate loss modeling!
Magnetic Circuit Modeling
Core Reluctance

\[ R_m = \frac{l_i}{\mu_0 \mu_r A_i} \]

Core Reluctance Dimensions

<table>
<thead>
<tr>
<th>Section</th>
<th>( l_i )</th>
<th>( A_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>( b )</td>
<td>( c \cdot t )</td>
</tr>
<tr>
<td>II</td>
<td>( d )</td>
<td>( a \cdot t )</td>
</tr>
<tr>
<td>III</td>
<td>( \frac{2\pi}{4} \cdot \frac{(a+c)}{4} = \frac{\pi}{8} (a + c) \cdot \frac{t(a+c)}{2} )</td>
<td></td>
</tr>
</tbody>
</table>

Mean magnetic length
Mean magnetic cross-sectional area
Magnetic Circuit Modeling
Air Gap Reluctance: Different Approaches (1)

Assumption of Homogeneous Field Distribution

\[
R_m = \frac{l_g}{\mu_0 A_g}
\]

- \(l_g\): Air gap length
- \(A_g\): Air gap cross-sectional area

Increase of the Air Gap Cross-Sectional Area

E.g. [1] (for a cross section with dimension \(a \times t\)):

\[
R_m = \frac{l_g}{\mu_0 (a + l_g)(t + l_g)}
\]

Magnetic Circuit Modeling
Air Gap Reluctance: Different Approaches (2)

Schwarz-Christoffel Transformation

Transformation Equation $z(t)$

$$z(t) = \frac{i}{\pi} \left( 2 \ln \left( 1 + \sqrt{1 - t} \right) - \ln t - 2 \sqrt{1 - t} \right)$$

($z$ and $t$ are complex numbers)

Transformation Equation $v(t)$

$$v(t) = \frac{V}{\pi} \ln t$$

Magnetic Circuit Modeling

Air Gap Reluctance: Different Approaches (3)

Solution to 2-D problems found in literature, e.g. in [3]

Can’t be directly applied to 3-D problems.

Some 3-D solution to problem found in literature; however, they are complex [4] and/or limited to one air gap shape [5]

More simple and universal model desired.


Magnetic Circuit Modeling
Aim of New Model

Air gap reluctance calculation that

- considers the three dimensionality,

- is reasonable easy-to-handle,

- is capable of modeling different shapes of air gaps,

- while still achieving a high accuracy.

Illustration of Different Air Gap Shapes:
Basic Structure for the Air Gap Calculation (2-D) [3]

\[ R'_{\text{basic}} = \frac{1}{\mu_0 \left[ \frac{w}{2l} + \frac{2}{\pi} \left( 1 + \ln \frac{\pi h}{4l} \right) \right]} \]
Magnetic Circuit Modeling
New Model (2)

2-D (1)

Basic Structure for the Air Gap Calculation
Magnetic Circuit Modeling
New Model (3)

2-D (2)

Air Gap Type 1

Air Gap Type 2

Air Gap Type 3

Basic Structure for the Air Gap Calculation

\[ R'_{\text{basic}} = \frac{1}{\mu_0} \left[ \frac{w}{2l} + \frac{2}{\pi} \left( 1 + \ln \frac{\pi h}{4l} \right) \right] \]
Magnetic Circuit Modeling
New Model (4)

2D → 3D : Fringing Factor (1)

Illustrative Example

zy-plane

zx-plane

Air gap per unit length

$R'_{zy} \Rightarrow \sigma_y = \frac{R'_{zy}}{l_g} \frac{1}{\mu_0 a}$

"Idealized" air gap (no fringing flux)

$R'_{zx} \Rightarrow \sigma_x = \frac{R'_{zx}}{l_g} \frac{1}{\mu_0 t}$
Illustrative Example

3-D Fringing Factor:

\[ \sigma = \sigma_x \sigma_y \]

\[ R_g = \sigma \frac{l_g}{\mu_0 at} \]

“Idealized” air gap (no fringing flux)

Alternative Interpretation:
Increase of air gap cross sectional area

\[ \frac{1}{\sigma_x} \quad \frac{1}{\sigma_y} \]

Magnetic Circuit Modeling
FEM Results

3-D FEM Simulation

Modeled Example

\[ a = 40 \text{ mm}; \ h = 40 \text{ mm} \]

Results

Graph showing
- (homogeneous field distribution)
- (increase of \( A_g \) [1])
- (new)
Magnetic Circuit Modeling
Experimental Results

Inductance Calculation
EPCOS E55/28/21, $N = 80$

\[
\begin{align*}
\text{TABLE I} & \\
\text{MEASUREMENT RESULTS OF E-CORE} & \\
\text{Air Gap Length} & \text{Calculated classically (3)} & \text{Calculated with new approach (12)} & \text{Measured} \\
1.0 \text{ mm} & 1.42 \text{ mH} & 1.97 \text{ mH} & 2.07 \text{ mH} \\
1.5 \text{ mm} & 0.96 \text{ mH} & 1.47 \text{ mH} & 1.58 \text{ mH} \\
2.0 \text{ mm} & 0.72 \text{ mH} & 1.22 \text{ mH} & 1.26 \text{ mH} \\
\end{align*}
\]

Saturation Calculation
EPCOS E55/28/21, $N = 80$

\[ l_g = 1 \text{ mm}, B_{sat} = 0.45 \text{ T} \]

\[
\begin{align*}
\text{TABLE II} & \\
\text{MEASUREMENT RESULTS OF E-CORE} & \\
\text{Calculated classically (3)} & \text{Calculated with new approach (12)} \\
L & 2.75 \text{ mH} & 3.55 \text{ mH} \\
I_{sat} & 4.6 \text{ A} & 3.6 \text{ A} \\
\end{align*}
\]

Measurement
$I_{sat} = 3.7 \text{ A}$
Flux and Reluctance Calculation

$$\phi = f(R_m(\phi), I)$$

This equation must be solved iteratively by using a numerical solving method, e.g. the Newton’s method.

Inductance Calculation

$$L = \frac{N^2}{R_{tot}(I)}$$

Reluctance Model

$$R_m = f(\phi) \quad \phi = f(R_m(\phi), I) = f(\phi, I)$$
Aim
Design PFC rectifier system.
Show trade-off between losses and volume.
Illustrative example.

Modeling of boost inductors (three individual inductors $L_{2a} = L_{2b} = L_{2c}$) will be step-by-step illustrated in the course of this presentation.
Example
Reluctance Model (1)

Photo & Dimensions

Dimensions
- $l_s$: 1.95 mm
- $d_0$: 60 mm
- $b$: 60 mm
- $a$: 20 mm
- $t$: 28 mm
- $ww$: 56.9 mm
- $d$: 2.24 mm

Material
Grain-oriented steel (M165-35S)

Calculation of Core Reluctances

$$R_{c1} = \frac{d_0 - 2a}{\mu_0 \mu_i at} + 2 \frac{\pi (2a)}{8 \mu_0 \mu_i t \frac{(2a)}{2}}$$

$$= \frac{60\text{mm} - 2 \cdot 20\text{mm}}{\mu_0 \cdot 20'000 \cdot 20\text{mm} \cdot 28\text{mm}} + 2 \frac{\pi (2 \cdot 20\text{mm})}{8 \mu_0 \cdot 20'000 \cdot 28\text{mm} \frac{(2 \cdot 20\text{mm})}{2}} = 3654 \frac{\text{A}}{\text{Vs}}$$

$$R_{c3} = \frac{d_0 - 2a + 2b}{\mu_0 \mu_i at} + 2 \frac{\pi (2a)}{8 \mu_0 \mu_i t \frac{(2a)}{2}}$$

$$= \frac{60\text{mm} - 2 \cdot 20\text{mm} + 2 \cdot h}{\mu_0 \cdot 20'000 \cdot 20\text{mm} \cdot 28\text{mm}} + 2 \frac{\pi (2 \cdot 20\text{mm})}{8 \mu_0 \cdot 20'000 \cdot 28\text{mm} \frac{(2 \cdot 20\text{mm})}{2}} = 12184 \frac{\text{A}}{\text{Vs}}$$

Reluctance Model
Example
Reluctance Model (2)

Calculation Air Gap Reluctances

zy-plane

\[
R_{zy,1} = \mu_0 \left[ \frac{a}{2l_g} + \frac{2}{\pi} \left( 1 + \ln \frac{\pi a}{4l_g} \right) \right]
\]

\[
R_{zy,2} = \mu_0 \left[ \frac{a}{2l_g} + \frac{2}{\pi} \left( 1 + \ln \frac{\pi (b + a)}{4l_g} \right) \right]
\]

\[
R_{zy,3} = \mu_0 \left[ \frac{a}{2l_g} + \frac{2}{\pi} \left( 1 + \ln \frac{\pi b}{4l_g} \right) \right]
\]

zx-plane

\[
R_{zx,1} = \mu_0 \left[ \frac{t}{2l_g} + \frac{2}{\pi} \left( 1 + \ln \frac{\pi a}{4l_g} \right) \right]
\]

\[
R_{zx,2} = \mu_0 \left[ \frac{t}{2l_g} + \frac{2}{\pi} \left( 1 + \ln \frac{\pi (b + a)}{4l_g} \right) \right]
\]

\[
R_{zx,3} = \frac{R'_{zx,1} + R'_{zx,2}}{2}
\]

Basic Reluctance

Material
Grain-oriented steel (M165-35S)

Inductance

\[
R_g = \sigma_x \sigma_y \frac{l_g}{\mu_0 at} = 1.66 \text{ MA/Wb}
\]

\[
L = \frac{N^2}{R_{c1} + R_{c2} + R_{g1} + R_{g2}} = 2.66 \text{ mH}
\]

meas.
2.69 mH
Outline

- Magnetic Circuit Modeling
- Core Loss Modeling
- Winding Loss Modeling
- Thermal Modeling
- Multi-Objective Optimization
- Summary & Conclusion
Core Loss Modeling
Overview of Different Core Materials (1)

- Iron Based Alloys - ferromagnetics -
  Alloys with some amount of Si, Ni, Cr or Co.
  Low electric resistivity.
  High saturation flux density.

- Ferrites - ferrimagnetics -
  Ceramic materials, oxide mixtures of iron and Mn, Zn, Ni or Co.
  High electric resistivity.
  Small saturation flux density compared to ferromagnetics.

- Laminated Cores
  Laminations are electrically isolated to reduce eddy currents.

- Powder Iron Cores
  Consist of small iron particles, which are electrically isolated to each other.

- Amorphous Alloys
  Made of alloys without crystalline order (cf. glasses, liquids).

- Nanocrystalline Materials
  Ultra fine grain FeSi, which are embedded in an amorphous minority phase.
## Core Loss Modeling

### Overview of Different Core Materials (2)

<table>
<thead>
<tr>
<th>Selection Criteria</th>
<th>Ferrite</th>
<th>Powder Iron Core</th>
</tr>
</thead>
<tbody>
<tr>
<td>Saturation Flux Density</td>
<td>Low Sat. Flux Density (0.45 T)</td>
<td>High Sat. Flux Density (1.5 T)</td>
</tr>
<tr>
<td>Power Loss Density</td>
<td>Low Losses</td>
<td>Moderate Losses</td>
</tr>
<tr>
<td>(Frequency Range)</td>
<td>Low Price</td>
<td>Low Price</td>
</tr>
<tr>
<td>Price</td>
<td>Many Different Shapes</td>
<td>Many Different Shapes</td>
</tr>
<tr>
<td>etc.</td>
<td>Very Brittle</td>
<td>Distributed Air Gap (low rel. permeability)</td>
</tr>
</tbody>
</table>

### Laminated Steel Cores
- Very High Sat. Flux Density (2.2T)
- High Losses
- Low Price
- Many Different Shapes

### Amorphous Alloys
- High Sat. Flux Density (1.5T)
- Low Losses
- High Price
- Limited Available Shapes

### Nanocrystalline Materials
- High Sat. Flux Density (1.1T)
- Very Low Losses
- Very High Price
- Limited Available Shapes

### Powder Iron Core
- High Sat. Flux Density (1.5 T)
- Moderate Losses
- Low Price
- Many Different Shapes
- Distributed Air Gap (low rel. permeability)

### Selection Criteria
- Saturation Flux Density
- Power Loss Density
- (Frequency Range)
- Price
- etc.

### Formulas
- **LF**: \( B_{\text{SAT}} = B_{\text{max}} \).
- **HF**: \( B_{\text{max}} \) is limited by core losses.
Core Loss Modeling
Overview of Different Core Materials (3)

Core Loss Modeling
Physical Origin of Core Losses (1)

Weiss Domains / Domain Walls

- Spontaneous magnetization.
- Material is divided to saturated domains (Weiss domains).
- In case an external field is applied, the domain walls are shifted or the magnetic moments within the domains change their direction. → The net magnetization becomes greater than zero.

- The flux change is partly irreversible, i.e. energy is dissipated as heat.
- The reason for this are the so called Barkhausen jumps, that lead to local eddy current losses.
- In case the loop is traversed very slowly, these Barkhausen jumps lead to the static hysteresis losses.
Core Loss Modeling
Physical Origin of Core Losses (2)

*B-H-Loop*

- If the process would be fully reversible, going from $B_1$ to $B_2$ would store potential energy in the magnetic material that is later released (i.e. the area of the closed loop would be zero).

- Since the process is partly irreversible, the area of the closed loop represents the energy loss per cycle

$$W = \oint H dB$$
Core Loss Modeling
Classification of Losses (1)

- (Static) hysteresis loss
  - Rate-independent \(BH\) Loop.
  - Loss energy per cycle is constant.
  - Irreversible changes each within a small region of the lattice (Barkhausen jumps).
  - These rapid, irreversible changes are produced by relatively strong local fields within the material.

- Eddy current losses

- Residual Losses – Relaxation losses
Core Loss Modeling
Classification of Losses (2)

- (Static) hysteresis loss

- Eddy current losses
  - Depend on material conductivity and core shape.
  - Affect $BH$ loop.

- Residual Losses – Relaxation losses

Measurements
VITROPERM 500F
Core Loss Modeling
Classification of Losses (3)

- (Static) hysteresis loss
- Eddy current losses
- Residual losses – Relaxation losses
  - Rate-dependent $BH$ Loop.
  - Reestablishment of a thermal equilibrium is governed by relaxation processes.
  - Restricted domain wall motion.
Core Loss Modeling

Typical Flux Waveforms

Sinusoidal

Triangular

e.g. 50/60 Hz isolation transformer

e.g. Buck / Boost converter

Trapezoidal

e.g. boost inductor of Dual Active Bridge

Combination

e.g. Boost inductor in PFC
Core Loss Modeling
Outline of Different Modeling Approaches

Steinmetz Approach

\[ P = k f^\alpha B^\beta \]

- Simple
- Steinmetz parameter are valid only in a limited flux density and frequency range
- DC Bias not considered
- (Only for sinusoidal flux waveforms)

Loss Separation

\[ P = P_{\text{hyst}} + P_{\text{eddy}} + P_{\text{residual}} \]

- Needed parameters often unknown
- Model is widely applicable
- Increases physical understanding of loss mechanisms

Loss Map Approach

(Loss Database)

- Measuring core losses is indispensable to overcome limits of Steinmetz approach

Hysteresis Model

(e.g. Preisach Model, Jiles-Atherton Model)

- Difficult to parameterize
- Increases physical understanding of loss mechanisms
Core Loss Modeling
Overview of Hybrid Modeling Approach

“The best of both worlds” (Steinmetz & Loss Map approach)

Outline of Discussion

- Derivation of the i²GSE. (1)
- How to measure core losses in order to build loss map. (2)
- Use of loss map. (3)
- How to calculate core losses for cores of different shapes? (4)

\[
P_v = \frac{1}{T} \int_0^T k_i \left( \frac{dB}{dt} \right)^\alpha (\Delta B)^{\beta-\alpha} dt + \sum_{i=1}^n Q_{ri} P_{ri} \rightarrow \frac{P}{V}
\]
Core Loss Modeling
Derivation of the $i^2$GSE – Motivation (1)

Steinmetz Equation SE

$$P_v = k f^\alpha \hat{B}^\beta$$

- Only sinusoidal waveforms (\(\rightarrow\) iGSE).

\[ P_v = \frac{1}{T} \int_{0}^{T} k_i \left| \frac{dB}{dt} \right|^\alpha (\Delta B)^{\beta-\alpha} dt \]

- DC bias not considered
- Relaxation effect not considered (\(\rightarrow\) i$^2$GSE)
- Steinmetz parameter are valid only for a limited flux density and frequency range

$P_v$: time-average power loss per unit volume

\[ k_i = k \frac{k}{(2\pi)^{\alpha-1} \int_{0}^{2\pi} |\cos \theta|^\alpha 2^{\beta-\alpha} d\theta} \]
Core Loss Modeling
Derivation of the i\(^2\)GSE – Motivation (2)

iGSE [8]

\[ P_v = \frac{1}{T} \int_0^T k_i \left| \frac{dB}{dt} \right|^\alpha (\Delta B)^{\beta-\alpha} dt \]

\[ k_i = \frac{k}{(2\pi)^{\alpha-1} \int_0^{2\pi} \cos^\alpha \theta 2^{\beta-\alpha} d\theta} \]

How to apply the formula?

Idea
- Generalized formula that is applicable for different flux waveforms
- Losses depend on dB/dt

For Sinusoidal Waveforms

\[ P_v = \frac{1}{T} \int_0^T k_i \left| \frac{dB}{dt} \right|^\alpha (\Delta B)^{\beta-\alpha} dt = k f^\alpha \left( \frac{\Delta B}{2} \right)^\beta \]

Core Loss Modeling
Derivation of the $i^2$GSE – Motivation (3)

Waveform

Results

iGSE

$$P_v = \frac{1}{T} \int_0^T k_i \left| \frac{dB}{dt} \right|^\alpha (\Delta B)^{\beta-\alpha} \, dt$$

Conclusion

Losses in the phase of constant flux!
Core Loss Modeling
Derivation of the $i^2$GSE – $B$-$H$-Loop

Relaxation Losses

- Rate-dependent $BH$ Loop.
- Reestablishment of a thermal equilibrium is governed by relaxation processes.
- Restricted domain wall motion.

Current Waveform
Core Loss Modeling
Derivation of the $i^2$GSE – Model Derivation 1

Waveform

Loss Energy per Cycle

Derivation (1)

Relaxation loss energy can be described with

$$E = \Delta E \left( 1 - e^{-\frac{t_1}{\tau}} \right)$$

$\tau$ is independent of operating point.

How to determine $\Delta E$?
Core Loss Modeling
Derivation of the $i^2$GSE – Model Derivation 1 (2)

$\Delta E$ – Measurements

**Waveform**

$\Delta E$ follows a power function!

**Conclusion**

$\Delta E = k_t \left| \frac{d}{dt} B(t) \right|^{\alpha_t} (\Delta B)^{\beta_t}$
Core Loss Modeling

Derivation of the $i^2$GSE – Model Derivation 1 (3)

Model Part 1

\[ P_v = \frac{1}{T} \int_{0}^{T} k_i \left| \frac{dB}{dt} \right|^\alpha (\Delta B)^{\beta-\alpha} \, dt + \sum_{l=1}^{n} P_{vl} \]

\[ P_{vl} = \frac{1}{T} k_r \left| \frac{d}{dt} B(t) \right|^\alpha_r (\Delta B)^{\beta_r} \left( 1 - e^{-\frac{t_l}{\tau}} \right) \]
Core Loss Modeling
Derivation of the $i^2$GSE – Model Derivation 2 (1)

**Explanation**

1) For values of $D$ close to 0 or close to 1 a loss underestimation is expected when calculating losses with iGSE (no relaxation losses included).

2) For values of $D$ close to 0.5 the iGSE is expected to be accurate.

3) Adding the relaxation term leads to the upper loss limit, while the iGSE represents the lower loss limit.

4) Losses are expected to be in between the two limits, as has been confirmed with measurements.
Core Loss Modeling
Derivation of the $i^2$GSE – Model Derivation 2 (2)

Waveform

Model Adaption

$$P_v = \frac{1}{T} \int_0^T k_i \left| \frac{dB}{dt} \right|^\alpha (\Delta B)^{\beta-\alpha} \, dt + \sum_{l=1}^n Q_{rl} \, P_{rl}$$

$Q_{rl}$ should be 1 for $D = 0$

$Q_{rl}$ should be 0 for $D = 0.5$

$Q_{rl}$ should be such that calculation fits a triangular waveform measurement.

$$Q_{rl} = e^{-q_t \frac{dB(t_+) / dt}{dB(t_-) / dt}} \left( = e^{-q_t \frac{D}{1-D}} \right)$$
Core Loss Modeling
Derivation of the $i^2\text{GSE} – \text{Model Derivation 2 (3)}$

Waveform

Power Loss

\[ P [W] \]

\[ D \]

$\text{i}^2\text{GSE}$

$\text{Upper Loss Limits}$

$\text{Measured Values}$
Core Loss Modeling
Derivation of the $i^2$GSE – Summary

The improved-improved Generalized Steinmetz Equation ($i^2$GSE) [9]

$$P_v = \frac{1}{T} \int_0^T k_i \left| \frac{dB}{dt} \right|^\alpha (\Delta B)^{\beta-\alpha} \, dt + \sum_{l=1}^{n} Q_{rl} P_{rl}$$

with

$$P_{rl} = \frac{1}{T} k_r \left| \frac{d}{dt} B(t) \right|^{\alpha_r} (\Delta B)^{\beta_r} \left( 1 - e^{\frac{-t_1}{\tau}} \right)$$

and

$$Q_{rl} = e^{-q_r \left| \frac{dB(t+)/dt}{dB(t-/dt)} \right|}$$

Core Loss Modeling
Derivation of the $i^2$GSE – Example

$i^2$GSE

Evaluated for each piecewise-linear flux segment

\[ P_v = \frac{1}{T} \int_0^T k_i \left| \frac{dB}{dt} \right|^\alpha (\Delta B)^{\beta-\alpha} \, dt + \sum_{l=1}^n Q_{rl} P_{rl} \]

Example

Evaluated for each voltage step, i.e. for each corner point in a piecewise-linear flux waveform.

\[ \frac{dB}{dt} = \begin{cases} \frac{\Delta B}{T/2-t_\gamma} & \text{for } t \geq 0 \text{ and } t < T/2 - t_\gamma \\ 0 & \text{for } t \geq T/2 - t_\gamma \text{ and } t < T/2 \\ -\frac{\Delta B}{T/2-t_\gamma} & \text{for } t \geq T/2 \text{ and } t < T - t_\gamma \\ 0 & \text{for } t \geq T - t_\gamma \text{ and } t < T \end{cases} \]

\[ P_v = \frac{T-2t_\gamma}{T} k_i \left| \frac{\Delta B}{T/2-t_\gamma} \right|^\alpha (\Delta B)^{\beta-\alpha} + \sum_{l=1}^2 Q_{rl} P_{rl} \]

with \( Q_{r1} = Q_{r2} = 0 \)

\[ P_{r1} = P_{r2} = \frac{1}{T} k_r \left| \frac{\Delta B}{T/2-t_\gamma} \right|^\alpha \left( 1 - e^{-\frac{t_\gamma}{\tau}} \right) \]

\[ \Delta B \]

\[ B_L \]

\[ t \]

\[ l = 1 \]

\[ l = 2 \]

\[ t_\gamma \]

\[ T \]

\[ k_i \]

\[ k_r \]

\[ \tau \]
Core Loss Modeling
Derivation of the $i^2$GSE – Conclusion

$P_v = \frac{1}{T} \sum_{i}^{T} \int_{0}^{T} k_i \frac{dB}{dt}^{\alpha} (\Delta B)^{\beta-\alpha} dt + \sum_{l=1}^{n} Q_{vl} P_{vl}$

Evaluated for each piecewise-linear flux segment

Evaluated for each voltage step, i.e. for each corner point in a piecewise-linear flux waveform.

Remaining Problems

Steinmetz parameter are valid only in a limited flux density and frequency range.

Core Losses vary under DC bias condition.

Modeling relaxation and DC bias effects need parameters that are not given by core material manufacturers.

Measuring core losses is indispensable!
Core Loss Modeling
Overview of Hybrid Modeling Approach

“The best of both worlds” (Steinmetz & Loss Map approach)

Outline of Discussion

- Derivation of the $i^2$GSE. (1)
- How to measure core losses in order to build loss map. (2)
- Use of loss map. (3)
- How to calculate core losses for cores of different shapes? (4)

$$P_v = \frac{1}{T} \int_0^T \left| k_i \frac{dB}{dt} \right|^{\alpha} (\Delta B)^{\beta-\alpha} dt + \sum_{l=1}^n Q_{rl} P_{rl} \quad (1)$$

$$P \quad (4)$$
Core Loss Modeling
Core Loss Measurement – Measurement Principle

Waveforms

- Sinusoidal
- Triangular
- Trapezoidal

Excitation System

- Voltage: 0 … 450 V
- Current: 0 … 25 A
- Frequency: 0 … 200 kHz

Schematic

Loss Extraction

\[
B(t) = \frac{1}{N_2 \cdot A_e} \int_0^t u(\tau) \, d\tau
\]

\[
H(t) = \frac{N_1 \cdot i(t)}{l_e}
\]

\[
\frac{P[W]}{V[m^3]} = f \int H dB
\]
Core Loss Modeling
Core Loss Measurement - Overview

System Overview

\[
\frac{P}{V} = f \int_0^T i_1(t) \frac{N_1}{N_2} v_2(t) \, dt
\]

\[
= f \int_0^T \frac{H(t)l_e}{N_1} \frac{N_1}{N_2} A_e \frac{dB(t)}{dt} \, dt
\]

\[
= f \int_{B(0)}^{B(T)} H(B) \, dB = f \int H dB
\]
Core Loss Modeling
Overview of Hybrid Modeling Approach

“The best of both worlds” (Steinmetz & Loss Map approach)

Outline of Discussion

- Derivation of the $i^2$GSE. (1)
- How to measure core losses in order to build loss map. (2)
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- How to calculate core losses for cores of different shapes? (4)

\[ P_v = \frac{1}{T} \int_0^T k_i \left| \frac{dB}{dt} \right|^\alpha (\Delta B)^{\beta-\alpha} dt + \sum_{l=1}^n Q_i l P_{vl} \]
Core Loss Modeling
Needed Loss Map Structure

Typical flux waveform

Content of Loss Map

Relaxation
\[ \alpha, \beta, k, r, q \]

\[ B-H \text{-Relation} \]

LF

HF

\[ \frac{\Delta B}{F}, F, H, V \text{, and } E \]
Core Loss Modeling
Minor and Major Loops

Idea

Losses due to Minor and Major Loops are calculated independent of each other and summed up.

Implementation

Actually, it is not considered how the minor loop closes: each piecewise linear segment is modeled as having half the losses of its corresponding closed loop (cf. next slides).

\[
\frac{P}{V} = \left(\frac{P}{V}\right)_{\text{Major}} + \sum \left(\frac{P}{V}\right)_{\text{Piecewise Linear Segment (+ Turning Point)}}
\]
Core Loss Modeling

Hybrid Loss Modeling Approach (1)
Core Loss Modeling

Hybrid Loss Modeling Approach (2)

Loss Map

These equations arise when one evaluates the $i^2$GSE for symmetric triangular waveforms.

Three loss map operating points are required in order to extract the parameters $\alpha$, $\beta$, and $k$ (or $k_i$).

Evaluated for the according corner point in a piecewise-linear flux waveform.

Evaluated for the according piecewise-linear flux segment.

$$P_v = \frac{1}{T} \int_0^T k_i \left| \frac{dB}{dt} \right|^\alpha (\Delta B)^{\beta - \alpha} dt + \sum_{l=1}^n Q_{rl} P_{rl}$$

---

(I)

(II)

$P_v = k_i (2f_1)^\alpha (\Delta B_1)^\beta$

$P_v = k_i (2f_2)^\alpha (\Delta B_2)^\beta$

$P_v = k_i (2f_3)^\alpha (\Delta B_3)^\beta$
Core Loss Modeling
Hybrid Loss Modeling Approach (3)

Interpolation and Extrapolation

\((H_{DC}^*, T^*, \Delta B^*, f^*)\)

**Diagram:**

- \(H_{DC}\) and \(T\)
- \(\Delta B\) and \(f\)
Core Loss Modeling
Hybrid Loss Modeling Approach (4)

Advantages of Hybrid Approach (Loss Map and $i^2GSE$):

- **Relaxation effects** are considered ($i^2GSE$).

- A **good interpolation** and extrapolation between premeasured operating points is achieved.

- Loss map provides accurate $i^2GSE$ parameters for a **wide frequency and flux density range**.

- A **DC bias is considered** as the loss map stores premeasured operating points at different DC bias levels.
Core Losses
Summary of Loss Density Calculation

\[
P = k f^\alpha B^\beta
\]

\[
P_v = \frac{1}{T} \int_0^T k_i \left| \frac{dB}{dt} \right|^\alpha (\Delta B)^{\beta-\alpha} dt + \sum_{l=1}^n Q_{rl} P_{rl}
\]
with \( Q \geq 0, n = 1 \)

\[
P_v = \frac{1}{T} \int_0^T k_i \left| \frac{dB}{dt} \right|^\alpha (\Delta B)^{\beta-\alpha} dt + \sum_{l=1}^n Q_{rl} P_{rl}
\]
with \( Q = 1, n = 2 \)

\[
P_v = \frac{1}{T} \int_0^T k_i \left| \frac{dB}{dt} \right|^\alpha (\Delta B)^{\beta-\alpha} dt + \sum_{l=1}^n Q_{rl} P_{rl}
\]
with different \( Q \)'s and \( n > 0 \).
Core Loss Modeling
Overview of Hybrid Modeling Approach

“The best of both worlds” (Steinmetz & Loss Map approach)

Outline of Discussion

- Derivation of the $i^2$GSE. (1)
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\[ P_v = \frac{1}{T} \int_0^T k_i \left[ \frac{dB}{dt} \right]^\alpha (\Delta B)^{\beta-\alpha} dt + \sum_{l=1}^n Q_{rl} P_{rl} \]  

\[ P = \frac{P}{V} \]
Core Loss Modeling
Effect of Core Shape

Procedure

1) The flux density in every core section of (approximately) homogenous flux density is calculated.

2) The losses of each section are calculated.

3) The core losses of each section are then summed-up to obtain the total core losses.

\[ \Phi = f(R_m(\Phi), I) = f(\Phi, I) \]
Core Loss Modeling
Effective Core Dimensions of Toroid

Motivation for Effective Core Dimensions
Core loss densities are needed to model core losses. It is difficult to determine these loss densities from a toroid, since the flux density is not distributed homogeneously in a toroid.

Definition: Ideal Toroid
A toroid is ideal when he has a homogenous flux density distribution over the radius \((r_1 \simeq r_2)\).

Idea for Real Toroid
Find effective core magnetic length and cross section, so one can calculate as if it were an ideal toroid, i.e. as if the flux density distribution were homogenous.

Effective magnetic length
\[
l_e = \frac{2\pi \ln \frac{r_2}{r_1}}{1/r_1 - 1/r_2}
\]

Effective magnetic cross-section
\[
A_e = \frac{h \ln^2 \frac{r_2}{r_1}}{1/r_1 - 1/r_2}
\]
Core Loss Modeling
Impact of Core Shape on Eddy Current Losses

Eddy current loss density can be determined as \([5]\)

\[
P_{\text{eddy}} = \frac{(\pi \hat{B} f d)^2}{k_{\text{ec}} \rho}
\]

<table>
<thead>
<tr>
<th>Geometry</th>
<th>(k_{\text{ec}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>laminations of thickness (d)</td>
<td>6</td>
</tr>
<tr>
<td>cylinder of diameter (d)</td>
<td>16</td>
</tr>
<tr>
<td>sphere of diameter (d)</td>
<td>20</td>
</tr>
</tbody>
</table>

For a laminated core it is

\[
P_{\text{eddy}} = \frac{(\pi \hat{B} f d)^2}{6 \rho}
\]

→ The eddy current losses per unit volume depend not on the shape of the bulk material, but on the size and geometry of the insulated regions.

→ In case of laminated iron cores, it is still appropriate to calculate with core loss densities that have been measured on a sample core with a geometrically different bulk material, but with the same lamination or tape thickness.

Core Loss Modeling
Effect in Tape Wound Cores

Thin ribbons (approx. 20 µm)
Wound as toroid or as double C core.
Amorphous or nanocrystalline materials.

Losses in gapped tape wound cores higher than expected!
Core Loss Modeling
Effect in Tape Wound Cores - Cause 1: Interlamination Short Circuits

**Machining process**
Surface short circuits introduced by machining (particularly a problem in in-house production).

After treatment may reduce this effect. At ETH, a core was put in an 40% ferric chloride FeCl$_3$ solution after cutting, which substantially (more than 50%) decreased the core losses.
Core Loss Modeling
Effect in Tape Wound Cores - Cause 2: Orthogonal Flux Lines (1)

A flux orthogonal to the ribbons leads to very high eddy current losses!
Core Loss Modeling
Effect in Tape Wound Cores - Cause 2: Orthogonal Flux Lines (2)

An experiment that illustrates well the loss increase due to an orthogonal flux is given here.

Displacements

Horizontal Displacement

Vertical Displacement

Core Loss Results
Core Loss Modeling
Effect in Tape Wound Cores - Cause 2: Orthogonal Flux Lines (3)

Core loss increase due to leakage flux in transformers.

Measurement Set Up

Results

A higher load current leads to higher orthogonal flux!

Secondary Current (A)

Core Losses (W)

0 1 2 3 4

0 4 6 8 10 12

Fixed Voltage

Fixed Flux

Variable Load

Orthogonal Flux

Primary Winding

Secondary Winding

FEM

Gecko - Simulations AG
Jonas Mühlethaler
Core Loss Modeling
Effect in Tape Wound Cores - Cause 2: Orthogonal Flux Lines (4)

In [10] a core loss increase with increasing air gap length has been observed.

Figures from [10]

Example
Core Loss Modeling (1)

Photo & Dimensions

Flux Density Distribution

An approximately homogeneous flux density distribution inside the core.

Reluctance Model

Flux Density Waveform

GeckoMAGNETICS Presentation

Material
Grain-oriented steel (M165-35S)
Example
Core Loss Modeling (2)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input voltage AC $V_{\text{mains}}$</td>
<td>230 V</td>
</tr>
<tr>
<td>Mains frequency $f_{\text{mains}}$</td>
<td>50 Hz</td>
</tr>
<tr>
<td>DC Voltage $V_{\text{dc}}$</td>
<td>650 V</td>
</tr>
<tr>
<td>Load Current $I_L$</td>
<td>15.4 A</td>
</tr>
</tbody>
</table>

SiFe vs. Ferrite
2 kHz vs. 20 kHz

Modeling with GeckoMAGNETICS
2 kHz / \( T_{\text{max}} = 65 \, ^\circ \text{C} / L = 2.5 \, \text{mH} / \text{Solid Round Wires} \)

SiFe (M165-35S)  
Ferrite (N87)
Example Core Loss Modeling (4)

20 kHz / $T_{\text{max}} = 65^\circ \text{C} / L = 1 \text{ mH} / $Litz Wires

SiFe (M165-35S)

Ferrite (N87)

Therm. Limit
Example
Core Loss Modeling (5)
Outline

- Magnetic Circuit Modeling
- Core Loss Modeling
- Winding Loss Modeling
- Thermal Modeling
- Multi-Objective Optimization
- Summary & Conclusion
Winding Loss Modeling
Skin Effect (1)

\(H\)-field in conductor

Ampere’s Law

\[ \oint H \, dl = \iint J \, dA \]

Faraday’s Law

\[ \oint E \, dl = -\frac{d}{dt} \iint B \, dA \]

Induced Eddy Currents

\( J_{\text{eddy}} \)
Winding Loss Modeling
Skin Effect (2)

FEM Simulation

Current Distribution

Outer radius of conductor
Winding Loss Modeling
Skin Effect (3)

**Skin Depth** (where the current density has 1/e of surface value)

\[
\delta = \frac{1}{\sqrt{\pi \mu_0 \sigma f}}
\]

\[
I = I_1
\]

\[
I = 0
\]

Power Loss Increase with Frequency

\[
P_S = F_R(f) \cdot R_{DC} \cdot \hat{I}^2
\]

\[
F_R(f)
\]

\[
\xi = \frac{d}{\sqrt{2} \delta}
\]
Winding Loss Modeling
Skin Effect (4)

Current Distributions

Parallel-connected (not twisted) conductors!

Figure from [19]
Winding Loss Modeling
Proximity Effect (1)

$H$-field of neighboring conductor induces eddy currents

Ampere’s Law

$$\oint H\,dl = \iint J\,dA$$

Faraday’s Law

$$\oint E\,dl = -\frac{d}{dt} \iint B\,dA$$
Winding Loss Modeling
Proximity Effect (2)

Eddy Currents in Conductor

Induced Eddy Currents

Current Concentration

Figures from [19]

\( H_{e,\text{rms}} = 35 \, \text{A/m parallel to conductor} \)
Winding Loss Modeling
Skin vs. Proximity Effect

Situation

\( f = 100 \text{ kHz}, \ I_{\text{peak}} = 1 \text{ A}, \ H_{\text{e,peak}} = 1000 \text{ A/m} \)

---

**Definition**

**Skin Effect Losses** \( P_{\text{Skin}} \)
Losses due to current \( I \), including loss increase due to self-induced eddy currents.

**Proximity Effect Losses** \( P_{\text{Prox}} \)
Losses due to eddy currents induced by external magnetic field \( H_{\text{e}} \).
Winding Loss Modeling
Litz Wire (1) - What are Litz wires?

Idea

\[ A_1 \rightarrow A_1/8 \]

Advantages of Litz wires
HF losses can be reduced substantially

Disadvantages of Litz wires
High price
Heat dissipation difficult
Higher \( R_{DC} \)

Implementation

www.wikipedia.org
Winding Loss Modeling
Litz Wire (2) - Why Litz Wires Have to be Twisted? (1)

Bundle-Level Skin Effect

\[
\oint Hdl = \iint JdA \\
\text{(Ampere’s Law)}
\]

\[
\oint Edl = -\frac{d}{dt} \iint BdA \\
\text{(Faraday’s Law)}
\]

Current Distributions

Internal Proximity Effect (will be explained later)
Winding Loss Modeling
Litz Wire (3) - Why Litz Wires Have to be Twisted? (2)

Bundle-Level Proximity Effect

\[ \oint H dl = \iint J dA \]  
(Ampere’s Law)

\[ \oint E dl = - \frac{d}{dt} \iiint B dA \]  
(Faraday’s Law)

Figures from [19]
Winding Loss Modeling
Litz Wire (4) – Strand-Level Effects

Internal and External Fields lead to Internal and External Proximity Effects

Skin effect
Internal prox. effect
Ext. prox. effect

Losses in Litz Wires

\[ (25 \times d_i = 0.5 \text{ mm}, \ I_{\text{peak}} = 5 \text{ A}, \ H_{e,\text{peak}} = 300 \text{ A/m}) \]

Losses in Solid Wires

\[ (d = 2.5 \text{ mm}, \ I_{\text{peak}} = 5 \text{ A}, \ H_{e,\text{peak}} = 300 \text{ A/m}) \]
## Winding Loss Modeling

### Litz Wire (5) – Types of Eddy-Current Effects in Litz Wire

<table>
<thead>
<tr>
<th></th>
<th>Strand-level</th>
<th>Bundle-level</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Skin Effect</strong></td>
<td><img src="image" alt="Skin Effect" /></td>
<td><img src="image" alt="Skin Effect" /></td>
</tr>
<tr>
<td><strong>Proximity Effect</strong></td>
<td><img src="image" alt="Proximity Effect" /></td>
<td><img src="image" alt="Proximity Effect" /></td>
</tr>
<tr>
<td>“Internal”</td>
<td><img src="image" alt="“Internal”" /></td>
<td><img src="image" alt="“Internal”" /></td>
</tr>
<tr>
<td>“External”</td>
<td><img src="image" alt="“External”" /></td>
<td><img src="image" alt="“External”" /></td>
</tr>
</tbody>
</table>

Winding Loss Modeling
Litz Wire (6) – Real Litz Wire

How do “real” Litz wires behave? [12]

Skin Effect / Internal Proximity Effect

\[ R_{\text{skin,\lambda}} = \lambda_{\text{skin}} R_{\text{skin,ideal}} + (1 - \lambda_{\text{skin}}) R_{\text{skin,parallel}} \]

External Proximity Effect

\[ R_{\text{prox,\lambda}} = \lambda_{\text{prox}} R_{\text{prox,ideal}} + (1 - \lambda_{\text{prox}}) R_{\text{prox,parallel}} \]

Litz Wire Type 1: 7 bundles with 35 strands each: \[ \lambda_{\text{skin}} \approx 0.5 / \lambda_{\text{prox}} \approx 0.99 \]
Litz Wire Type 2: 4 bundles with 61/62 strands each: \[ \lambda_{\text{skin}} \approx 0.9 / \lambda_{\text{prox}} \approx 0.99 \]

Winding Loss Modeling
Litz Wire (7) - Are Litz Wires Better than Solid Conductors?

Skin and Internal Proximity Effect

External Proximity Effect
Winding Loss Modeling
Foil Windings Enclosed by Magnetic Material

Advantages of foil windings
- HF losses can be reduced
- Lower price compared to Litz wire
- High filling factor

Disadvantages of foil windings
- Increased winding capacitance
- Risk of orthogonal flux

“Skin” of foil conductor larger than of round conductor with same cross section; hence, skin effect losses lower in foil conductor.
Winding Loss Modeling
Foil Windings Not Enclosed by Magnetic Material (1)

Orthogonal flux leads to increased skin and proximity effect.
Winding Loss Modeling
Foil Windings Not Enclosed by Magnetic Material (2)

(Foil) Windings with Return Conductors

$P$ [mW] vs. $f$ [kHz]

$I_{peak} = 1$ A

$d = 1.95$ mm

$l = 1$ m

10 mm x 0.3 mm

FEM @ 8 kHz
Winding Loss Modeling
Overview About Different Winding Types

<table>
<thead>
<tr>
<th>Type</th>
<th>Price</th>
<th>Skin &amp; Proximity</th>
<th>Filling Factor</th>
<th>Heat Dissipation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Round Solid Wire</td>
<td>++</td>
<td>--</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Litz Wire</td>
<td>--</td>
<td>++</td>
<td>-</td>
<td>--</td>
</tr>
<tr>
<td>Foil Winding</td>
<td>+</td>
<td>+</td>
<td>++</td>
<td>+</td>
</tr>
<tr>
<td>Rectangular Wire</td>
<td>++</td>
<td>-</td>
<td>++</td>
<td>+</td>
</tr>
</tbody>
</table>

Table and Figure from [13] M. Albach, “Induktive Komponenten in der Leistungselektronik”, VDE Fachtagung - ETG Fachbereich Q1 "Leistungselektronik und Systemintegration", Bad Nauheim, 14.04.2011
Winding Loss Modeling
Skin Effect of Foil Conductor

Geometry Considered

\[ P_S = F_F(f) \cdot R_{DC} \cdot \hat{I}^2 \]
(Loss per unit length)

with

\[ F_F = \frac{v \sinh v + \sin v}{4 \cosh v - \cos v} \]

\[ R_{DC} = \frac{1}{\sigma bh} \]

\[ v = \frac{h}{\delta} \]

\[ \delta = \frac{1}{\sqrt{\pi \mu_0 \sigma f}} \]

\( F_F \) evaluated

Current Distribution

![Current Density Graph](image)
Winding Loss Modeling
Proximity Effect of Foil Conductor

Geometry Considered

\[ P_P = G_F(f) \cdot R_{DC} \cdot \hat{H}_S^2 \]
(Loss per unit length)

with

\[ G_F = b^2 \frac{\sinh \nu - \sin \nu}{\cosh \nu + \cos \nu} \]
\[ R_{DC} = \frac{1}{\sigma bh} \]
\[ \nu = \frac{h}{\delta} \]
\[ \delta = \frac{1}{\sqrt{\pi \mu_0 \sigma f}} \]

Current Distribution

\( G_F \) evaluated

(b = 1 m)
Winding Loss Modeling
Skin Effect of Solid Round Conductor

\[ P_S = F_R(f) \cdot R_{DC} \cdot \hat{I}^2 \]

(Loss per unit length)

with

\[ R_{DC} = \frac{4}{\sigma \pi d^2} \]
\[ \xi = \frac{d}{\sqrt{2} \delta} \]
\[ \delta = \frac{1}{\sqrt{\pi \mu_0 \sigma f}} \]

\[ F_R = \frac{\xi}{4 \sqrt{2}} \left[ \frac{\text{ber}_0(\xi)\text{bei}_1(\xi) - \text{ber}_0(\xi)\text{ber}_1(\xi)}{\text{ber}_1(\xi)^2 + \text{bei}_1(\xi)^2} - \frac{\text{bei}_0(\xi)\text{ber}_1(\xi) - \text{bei}_0(\xi)\text{bei}_1(\xi)}{\text{ber}_1(\xi)^2 + \text{bei}_1(\xi)^2} \right] \]
Winding Loss Modeling
Proximity Effect of Solid Round Conductor

\[ P_{P} = G_{R}(f) \cdot R_{DC} \cdot \hat{H}_{S}^{2} \]

(Loss per unit length)

with

\[ R_{DC} = \frac{4}{\sigma \pi d^{2}} \]
\[ \xi = \frac{d}{\sqrt{2} \delta} \]
\[ \delta = \frac{1}{\sqrt{\pi \mu_{0} \sigma f}} \]
\[ G_{R} = -\frac{\xi \pi^{2} d^{2}}{2 \sqrt{2}} \left[ \frac{\text{ber}_{2}(\xi)\text{ber}_{1}(\xi) + \text{ber}_{2}(\xi)\text{bei}_{1}(\xi)}{\text{ber}_{0}(\xi)^{2} + \text{bei}_{0}(\xi)^{2}} + \frac{\text{bei}_{2}(\xi)\text{bei}_{1}(\xi) - \text{bei}_{2}(\xi)\text{ber}_{1}(\xi)}{\text{ber}_{0}(\xi)^{2} + \text{bei}_{0}(\xi)^{2}} \right] \]
Winding Loss Modeling
Skin and Proximity Effect of Litz Wire

Skin Effect

\[ P_S = n \cdot R_{DC} \cdot F_R (f) \cdot \left( \frac{\hat{I}}{n} \right)^2 \]

(Loss per unit length)

Losses in Litz Wires

Proximity Effect

\[ P_P = P_{P,e} + P_{P,i} \]

\[ = n \cdot R_{DC} \cdot G_R (f) \cdot \left( H_e^2 + \frac{\hat{I}^2}{2\pi^2 d_a^2} \right) \]

(Loss per unit length)

Average internal field \( H_i \) under the assumption of a homogeneous current distribution inside the Litz wire.

(25 x \( d_i \) = 0.5 mm, \( I_{\text{peak}} \) = 5 A, \( H_{e,\text{peak}} \) = 300 A/m)
Winding Loss Modeling
Orthogonality of Winding Losses

It is valid to calculate the losses for each frequency component independently and total them up.

\[ P = \sum_{i=0}^{\infty} \left( P_{S,i} + P_{P,i} \right) \]

It is valid to calculate the skin and proximity losses independently and total them up.

Winding Loss Modeling
Calculation of External Field $H_e$ (1D - Approach)

Un-Gapped Transformer Cores

\[ P = R_{DC} \left( F_{R/F} \hat{I}^2 NM + N G_{R/F} \sum_{m=1}^{M} \hat{H}_{avg,m}^2 \right) l_m \]

with

\[ H_{avg} = \frac{1}{2} \left( H_{left} + H_{right} \right) \]

it is

\[ P = R_{DC} \hat{I}^2 \left( F_{R/F} NM + N^3 MG_{R/F} \frac{4M^2 - 1}{12b_F^2} \right) l_m \]

where

$N$ … the number of conductors per layer
(i.e. $N = 1$ for foil windings)

$M$ … the number of layers.

\[ \oint H dl = \iint j dA \] (Ampere’s Law)
Winding Loss Modeling
Short Foil Conductors

“Porosity Factor”

\[ \eta = \frac{Nb_L}{b_F} \]

Redefinition of Parameters

\[ \sigma' = \eta \sigma \]

\[ \delta' = \frac{1}{\sqrt{\pi f \sigma' \mu_0}} \]

\[ \nu' = \frac{h}{\delta'} \]
Winding Loss Modeling
FEM Simulations : Foil Windings

Error < 6.5%

Error < 6.5%
Winding Loss Modeling
Calculation of External Field $H_e$ (2D - Approach)

Gapped cores: 2D approach is necessary!
Winding Loss Modeling
Effect of the Air Gap Fringing Field

The air gap is replaced by a fictitious current, which …

… has the value equal to the magneto-motive force (mmf) across the air gap.

\[ V_{m,\text{air}} = \oint_{P_1}^P \mathbf{H} \, \mathrm{d}l = \phi \cdot R_{\text{air}} \]

\[ V_{m,\text{air}} = \oint_{P_1}^P \mathbf{H} \, \mathrm{d}l = I \]

→ An accurate air gap reluctance model is needed!
Winding Loss Modeling
Effect of the Core Material

The method of images (mirroring)

“Pushing the walls away”
Winding Loss Modeling
Calculation of External Field $H_e$ (2D - Approach)

Gapped cores: 2D approach

External field vector across conductor $q_{xi;yk}$

\[
\hat{H}_e = \sum_{u=1}^{m} \sum_{l=1}^{n} \epsilon(u, l) \frac{i_{x_u,y_l} ((y_l - y_k) - j(x_u - x_i))}{2\pi ((x_u - x_i)^2 + (y_l - y_k)^2)}
\]
Winding Loss Modeling
Different Winding Sections

Section 1
Many mirroring steps necessary in order to push the walls away.

Section 2
Only one mirroring step necessary (only one wall).

→ Normally, higher proximity losses in Section 1.
Winding Loss Modeling

FEM Simulations: Round Windings (Including Litz Wire Windings) (1)

Major Simplification

- magnetic field of the induced eddy currents neglected.

- This can be problematic at frequencies above (rule-of-thumb) [15]

\[ f_{\text{max}} = \frac{2.56}{\pi \mu_0 \sigma d^2} \]

Results of considered winding arrangements

<table>
<thead>
<tr>
<th>f-range</th>
<th>( f &lt; f_{\text{max}} )</th>
<th>( f &gt; f_{\text{max}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error</td>
<td>&lt; 5%</td>
<td>&gt; 5% (always &lt; 25%)</td>
</tr>
</tbody>
</table>

Winding Loss Modeling
FEM Simulations: Round Windings (Including Litz Wire Windings) (2)

**FEM Simulation**

**Results**

Arrangement 1  $f = 20 \text{ kHz}$
Arrangement 2  $f = 10 \text{ kHz}$
Arrangement 3  $f = 100 \text{ kHz}$
Winding Loss Modeling
Methods to Decrease Winding Losses (1)

Interleaving

Optimal Solid Wire Thickness

Optimal Foil Thickness

Litz Wire

Avoid Orthogonal Flux in Foil Windings

Push this point to higher frequencies!

→ Increase number of strands.
(resp.: find optimal number of strands)
Winding Loss Modeling
Methods to Decrease Winding Losses (2)

Arrangement of Windings

Proximity losses increase in more compact winding arrangements.
Aluminum vs. Copper [13]

Aluminum (vs. Copper):
- Lighter
- Lower costs
- Lower Conductivity \( \sigma = 38 \cdot 10^6 \, 1/(\Omega m) \)
  (Copper: \( \sigma = 58 \cdot 10^6 \, 1/(\Omega m) \))

→ Lower Skin Depth!

Skin- and DC losses higher than in copper conductors.

Proximity losses are lower in aluminum conductors over a wide frequency range. Figure shows a comparison of single round solid conductors in external field.

"Leistungselektronik und Systemintegration", Bad Nauheim, 14.04.2011
Example
Winding Loss Modeling

Photo & Dimensions

Material
Grain-oriented steel (M165-35S)

Current Waveform

Demonstration in GeckoMAGNETICS

Grain-orientation

- oriented steel (M165-35S)

Dimensions

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l_x$</td>
<td>1.95 mm</td>
</tr>
<tr>
<td>$d_o$</td>
<td>60 mm</td>
</tr>
<tr>
<td>$t$</td>
<td>28 mm</td>
</tr>
<tr>
<td>$w_w$</td>
<td>56.9 mm</td>
</tr>
<tr>
<td>$d$</td>
<td>2.24 mm</td>
</tr>
</tbody>
</table>

Material
Grain-oriented steel (M165-35S)
Outline

- Magnetic Circuit Modeling
- Core Loss Modeling
- Winding Loss Modeling
- Thermal Modeling
- Multi-Objective Optimization
- Summary & Conclusion
Thermal Modeling
Motivation & Model (1)

Motivation
Thermal modeling is important to …

… avoid overheating.
… improve loss calculation (since the losses depend on temperature).
Thermal Modeling
Motivation & Model (2)

Model

→ Determination of thermal resistors is challenging!
Thermal Modeling
Heat Transfer Mechanisms

\[ R_{th} = \frac{\Delta T}{P} = f(T) \]

Conduction
Independent of temperature \( T \) for most materials
Difficult to determine interfaces between materials

\[ R_{th} = \frac{\Delta T}{P} = \frac{l}{A \lambda} \]

Convection
Combined effect of conduction and fluid flow
Changes with changing absolute temperature (nonlinear)
Good empirical calculation approach available

\[ R_{th} = \frac{\Delta T}{P} = \frac{1}{\alpha A} \]

Radiation
Small compared to other mechanisms
Modeling the system is demanding
(nonlinear eq. / to describe which components “sees” the other component).

\[ P = \varepsilon_{eff} A_1 \sigma (T_b^4 - T_a^4) \]
Thermal Modeling
Thermal Resistance Calculation: (Natural) Convection (1)

\[ R_{th} = \frac{\Delta T}{P} = \frac{1}{\alpha A} \]

\( \alpha \) is a coefficient that is influenced by …

… the absolute temperature,
… the fluid property,
… the flow rate of the fluid,
… the dimensions of the considered surface,
… orientation of the considered surface,
… and the surface texture.
Thermal Modeling
Thermal Resistance Calculation: (Natural) Convection (2)

Empirical solutions known for ...
Thermal Modeling
Thermal Resistance Calculation: (Natural) Convection (3)

Structure of Empirical Solutions - Theory

<table>
<thead>
<tr>
<th>Name</th>
<th>Measure of ...</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nusselt number</td>
<td>... improvement of heat transfer compared to the case with hypothetical static fluid.</td>
</tr>
<tr>
<td>Nu</td>
<td></td>
</tr>
<tr>
<td>Grashof number</td>
<td>... ration between buoyancy and frictional force of fluid.</td>
</tr>
<tr>
<td>Gr</td>
<td></td>
</tr>
<tr>
<td>Prandtl number</td>
<td>... ratio between viscosity and heat conductivity of fluid.</td>
</tr>
<tr>
<td>Pr</td>
<td></td>
</tr>
<tr>
<td>Rayleigh number</td>
<td>... flow condition (laminar or turbulent) of fluid.</td>
</tr>
<tr>
<td>Ra</td>
<td></td>
</tr>
</tbody>
</table>

\[
Nu = \frac{l}{A\lambda} \frac{1}{\alpha A} = \frac{al}{\lambda} = f(Gr, Pr)
\]

\[
Gr = \frac{gl^3}{v^3} \beta \Delta T
\]

\[
Pr = 0.7 \quad \text{(for air)}
\]

\[
Ra = Pr \cdot Gr
\]

\[
\lambda \text{ is the heat conductivity of the fluid}
\]

\[
\lambda_{air} = 25.873 \text{ mW/(m K) @ 20°C} [16]
\]

Thermal Modeling
Thermal Resistance Calculation: (Natural) Convection (4)

Structure of Empirical Solutions - Example

Vertical Plane

\[ \nuu = \left( 0.825 + 0.387 \left( Ra \cdot f_1(Pr) \right)^{1/6} \right)^2 \]

\[ f_1 = \left( 1 + \left( \frac{0.492}{Pr} \right)^{9/16} \right)^{-16/9} \]

Reference:

Example
\( (h = 10 \text{ cm}, T_p = 60^\circ \text{C}, T_a = 20^\circ \text{C}, A = h \cdot h) \)
\( \Rightarrow R_{th} = 16.6 \text{ K/W} \)

Increase of Winding Surface

\[ A_2 = A_1 \cdot \frac{\pi}{2} \]
Thermal Modeling
Thermal Resistance Calculation: Conduction

Overview

Normally, with natural convection it is

\[ R_{th,W} \ll R_{th,WA} \]

Round Conductor

\( R_{th,W} \) is difficult to determine. One difficulty is, e.g., to model the influence of pressure on the thermal resistance.

Foil Conductor

Litz wire shows low thermal conductivity.
Example
Thermal Modeling (1)

(1) Horizontal Plane - Top

\[ Gr = \frac{g l^3}{\nu^3} \beta \Delta T \]

\[ Pr = 0.7 \] (for air)

\[ Ra = Pr \cdot Gr \]

\[ Nu = \begin{cases} 
0.766 (Ra \cdot f_2(Pr))^{1/5} & \text{for } Ra \cdot f_2(Pr) \leq 7 \cdot 10^4 \quad \text{(laminar)} \\
0.15 (Ra \cdot f_2(Pr))^{1/3} & \text{for } Ra \cdot f_2(Pr) > 7 \cdot 10^4 \quad \text{(turbulent)}
\end{cases} \]

with

\[ f_2 = \left( 1 + \left( \frac{0.322}{Pr} \right) \right)^{11/20} \]

\[ \alpha = \frac{Nu(Gr, Pr) \lambda}{l} \]

\[ R_{th} = \frac{\Delta T}{P} = \frac{1}{\alpha A} \]

Resistor \( R_{th} \) is now calculated for one operating point! (\( \rightarrow \) more iterations are necessary!)
Example
Thermal Modeling (2)

(2) Core to Winding

Heat flow via mechanical attachment has not been considered.

\[ R_{\text{CF}} = \frac{l}{A\lambda} = 1.05 \, \frac{\text{K}}{\text{W}} \]

\[ R_{\text{W}} < R_{\text{WA}} \]

Drawing represents the cross-section of one coil former side (there are total 2 x 4 coil former sides).

Quantity Values

\[ R_{\text{th,CA}} \approx 17.4 \, \text{K/W} \]

\[ R_{\text{th,WA}} \approx 5.8 \, \text{K/W} \]
Outline

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Multi-Objective Optimization – Volume vs. Losses
Introduction to the PFC Rectifier

**Goal:** Design boost inductor such that the max. current ripple is 4A.
Multi-Objective Optimization – Volume vs. Losses
Boost Inductor

Selected Inductor Shape

Material
Grain-oriented steel (M165-35S, lam. thickness 0.35 mm)
Multi-Objective Optimization – Volume vs. Losses
Pareto-Optimized Design of Boost Inductor

Constraints concerning boost inductors

max. current ripple $I_{HF,pp,max} = 4 \text{ A}$
max. temperature $T_{\text{max}} = 120 \degree \text{C}$

Boost Inductance

$L_{2,\text{min}}$ can be calculated based on the constraint $I_{HF,pp,max}$. The maximum current ripple $I_{HF,pp,max}$ occurs when the fundamental current peaks.

\[
L_{2,\text{min}} = \frac{\sqrt{3} \sqrt{2} |V_{\text{mains}}|}{V_{\text{DC}}} \cos(\pi/6) \cdot \frac{2}{3} \frac{V_{\text{DC}} - \sqrt{2} V_{\text{mains}}}{I_{HF,pp,max} \cdot f_{\text{sw}}} \\
= 2.53 \text{ mH}
\]
Multi-Objective Optimization – Volume vs. Losses
Pareto-Optimized Design of Boost Inductor - Simplifications

Simplified current / voltage waveforms for optimization procedure

Expectations
Loss overestimation in $L_2$ expected.
Multi-Objective Optimization – Volume vs. Losses
Pareto-Optimized Design of Boost Inductor - Design Space in GeckoMAGNETICS

Start

\[ L_{\text{Boost}} = 2.53 \text{ mH} \]
\[ T_{\text{max}} = 120 \degree \text{C} \]

Core

UI 39 – UI 90 cores of DIN41302
Material M165-35S
gap 2.0 … 3.0 mm (by 0.25 mm)
70 – 90 stacked sheets (by 10)

Winding

Solid round wire with fill factor 0.3
Split windings
Multi-Objective Optimization – Volume vs. Losses
Pareto-Optimized Design of Boost Inductor - Waveform/Cooling in GeckoMAGNETICS

Waveform

\[
\text{Voltage Time Area} = \delta_{100} \cdot \frac{2}{3} \frac{V_{DC} - \sqrt{2}V_{\text{mains}}}{I_{\text{HF,pp,max}} \cdot f_{\text{sw}}} = \frac{\sqrt{3}}{6} \frac{V_{\text{mains}}}{V_{DC}} \cos \left( \frac{\pi}{6} \right) \cdot \frac{2}{3} \frac{V_{DC} - \sqrt{2}V_{\text{mains}}}{I_{\text{HF,pp,max}} \cdot f_{\text{sw}}} = 0.01 \text{ Vs}
\]

Cooling

Forced convection

\[\rightarrow 3 \text{ m/s}\]

\[I_{\text{LF,peak}} = 21.8 \text{ A}\]
Multi-Objective Optimization – Volume vs. Losses
Pareto-Optimized Design of Boost Inductor - Performing Calculation

Procedure

1) A reluctance model is introduced to describe the electric / magnetic interface, i.e. $L = f(i)$.

2) Core losses are calculated.

3) Winding losses are calculated.

4) Inductor temperature is calculated.

Considered effects

- Air gap stray field
- Non-linearity of core material
- DC Bias
- Different flux waveforms
- Wide range of flux densities and frequencies
- Skin and proximity effect
- Stray field proximity effect
- Effect of core to magnetic field distribution
Multi-Objective Optimization – Volume vs. Losses
Pareto-Optimized Design of Boost Inductor - Results

Filter Losses vs. Filter Volume
Pareto Front

Prototype built
Pareto-front
Multi-Objective Optimization – Volume vs. Losses
Detailed Modeling in GeckoMAGNETICS

GeckoCIRCUITS

GeckoMAGNETICS
Multi-Objective Optimization – Volume vs. Losses

Results

Simulated Current Waveform

\[ I_{\text{FE,pp,\text{max}}} = 4.5 \, \text{A} \]
\[ P_{\text{Loss}} = 45.9 \, \text{W} \]

Measured Current Waveform

\[ I_{\text{FE,pp,\text{max}}} = 4.7 \, \text{A} \]
\[ P_{\text{Loss}} = 46.8 \, \text{W} \]

Conclusion

Loss modeling very accurate.

Photo

Dimensions

\( N \) 94
\( a \) 2.0 mm
\( d_o \) 60 mm
\( h \) 60 mm
\( w \) 20 mm
\( t \) 28 mm
\( w_w \) 56.9 mm
\( d \) 2.24 mm
Outline

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Summary & Conclusion
Magnetic Circuit Modeling

Air Gap Reluctance Calculation

**zy-plane**

\[ R'_{zy} \Rightarrow \sigma_y = \frac{R'_{zy}}{l_g} \frac{1}{\mu_0 a} \]

**zx-plane**

\[ R'_{zx} \Rightarrow \sigma_x = \frac{R'_{zx}}{l_g} \frac{1}{\mu_0 t} \]

\[
R_g = \sigma_x \sigma_y \frac{l_g}{\mu_0 at}
\]
Summary & Conclusion
Core Loss Modeling

“The best of both worlds” (Steinmetz & Loss Map approach)

\[
P_v = \frac{1}{T} \int_0^T k_i \frac{dB}{dt} \alpha (\Delta B)^{\beta-\alpha} dt + \sum_{l=1}^n Q_{rl} P_{rl} \rightarrow \frac{P}{V}
\]
Summary & Conclusion
Winding Loss Modeling

Optimal Solid Wire Thickness

\( P_{\text{Total}} \)
\( P_{\text{Prox}} \)
\( P_{\text{Skin}} \)

\[ f = 100 \, \text{kHz}, \quad I_{\text{peak}} = 1 \, \text{A}, \quad H_{e,\text{peak}} = 1000 \, \text{A/m} \]

Losses in Litz Wires

Internal prox. effect
Skin effect
Ext. prox. effect

Foil vs. Round Conductors

\( I = 1 \, \text{m} \)
\( d = 1.95 \, \text{mm} \)
\( l = 1 \, \text{m} \)
\( I_{\text{peak}} = 1 \, \text{A} \)

Gapped cores: 2D approach
Summary & Conclusion
Magnetic Design Environment

Core Material Database
GeckoDB

Automated Measurement System

Magnetics Design Software
GeckoMAGNETICS

Prototype

Circuit Simulator
GeckoCIRCUITS

Verification
(e.g. with Calorimeter)
Summary & Conclusion
Multi-Objective Optimization

PFC Rectifier

Inductor Pareto Front
GeckoMAGNETICS

Save Time
Fast and accurate design of magnetic components
Easy-to-use for non-expert

Increase Flexibility
Tool shows more than one realization possibility
In-house design of magnetics crucial for optimal designs.

Most Loss Effects are Considered
Skin- and proximity losses in litz, round and foil windings, air gap stray field losses,
DC bias core losses, thermal model, ...

www.gecko-simulations.com
Additional References


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